1990-03

A computer simulation of a MAD buoy field

Pardue, Phillip C.

Monterey, California: Naval Postgraduate School

http://hdl.handle.net/10945/34856

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

A COMPUTER SIMULATION OF A MAD BUOY FIELD

by

Phillip C. Pardue

March, 1990

Thesis Advisor: R. N. Forrest

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution is unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b. OFFICE SYMBOL
55

7a. NAME OF MONITORING ORGANIZATION
Naval Postgraduate School

6c. ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943-5000

7b. ADDRESS (City, State, and ZIP Code)
Monterey, California 93943-5000

8a. NAME OF FUNDING/SPONSORING ORGANIZATION

8b. OFFICE SYMBOL

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State, and ZIP Code)

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.

11. TITLE (Including Security Classification)
A COMPUTER SIMULATION OF A MAD BUOY FIELD

12 PERSONAL AUTHOR(S)
Phillip C. Pardue

13 TYPE OF REPORT
Master's Thesis

13b. TIME COVERED
FROM TO

14. DATE OF REPORT (Year, Month, Day)
1990, March

15. Page Count
49

16. SUPPLEMENTAL NOTATION
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Magnetic Anomaly Detection, MAD, Detection Models, Search Models, MAD Buoy Computer

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
The thesis describes an analysis of a Magnetic Anomaly Detection (MAD) buoy field. The analysis was based on estimates of the probability a submarine would be detected during an encounter with a MAD buoy. The estimates were determined by using a simulation of both a crosscorrelation detection system and a square law detection system. Using these estimates, a random search model was used to determine a lower bound on the search effectiveness of a MAD buoy field. In doing this, the effects of false alarm rate, sweep width, noise level, buoy depth, and target displacement were analyzed. A discussion of the simulation inputs, as well as a program listing are included in the thesis in order to facilitate future use of the simulation. Keywords: Magnetic signals; input parameters. (kr)

20 DISTRIBUTION/AVAILABILTIY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC

21. ABSTRACT SECURITY CLASSIFICATION
Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL
R. N. Forrest

22b. TELEPHONE (Include Area Code)
(408)646-2653

22c. OFFICE SYMBOL
55Fo

DD Form 1473, JUN 86 Previous editions are obsolete.
S/N 0102-LF-014-6603

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

A Computer Simulation of a MAD Buoy Field

by

Phillip C. Pardue
Lieutenant, United States Navy
B.S., Vanderbilt University, 1983

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1990

Author: [redacted]
Phillip C. Pardue

Approved by: [redacted]
R. N. Forrest, Thesis Advisor

[redacted]
James N. Eagle, Second Reader

[redacted]
Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

The thesis describes an analysis of a Magnetic Anomaly Detection (MAD) buoy field. The analysis was based on estimates of the probability a submarine would be detected during an encounter with a MAD buoy. The estimates were determined by using a simulation of both a crosscorrelation detection system and a square law detection system. Using these estimates, a random search model was used to determine a lower bound on the search effectiveness of a MAD buoy field. In doing this, the effects of false alarm rate, sweep width, noise level, buoy depth, and target displacement were analyzed. A discussion of the simulation inputs, as well as a program listing are included in the thesis in order to facilitate future use of the simulation.

TABLE OF CONTENTS

I. INTRODUCTION

This thesis describes an analysis of a Magnetic Anomaly Detection (MAD) buoy field. The analysis is based on estimates of the probability a submarine would be detected during an encounter with a MAD buoy. The estimates were generated with a program that simulates an encounter between a magnetometer based detection system and a submarine. The program allows a choice of two detection system models: a crosscorrelation detection system model and a square law detection system model. In the simulation, an encounter is a straight line encounter with a constant vertical separation between the magnetometer and the submarine. Probability estimates generated by the crosscorrelation detection system simulations are considered to represent upper bounds while probability estimates generated with the square law detection system simulations are considered to represent lower bound.

The probability estimates generated with the simulation were used to estimate MAD buoy sweep widths. Probabilities of detection for a MAD buoy field were then estimated by using the sweep width estimates in a sonobuoy field random search model. Relative to operationally realizable values, probabilities estimated with the random search model are considered to provide lower bounds on the search effectiveness of a buoy field.

Chapter 2 contains a discussion of the two detection system models, and Chapter 3 contains a discussion of the input parameters used in the simulation. Chapter 4 describes a sensitivity analysis of the simulation in which the effects of changes in the noise level, buoy depth and, submarine displacement are estimated.

The computer simulation was implemented using Microsoft Quick Basic version 4.5 on an IBM compatible 80286 computer. A program listing is contained in Appendix A.

II. MODEL DESCRIPTION

The encounter model and the computer program that was used as a basis for this thesis are described in Reference 1.

A. THE MAGNETIC SIGNAL

The encounter model is based on the following assumptions: "a submarine magnetic anomaly field is a magnetic dipole field that is superimposed on an earth magnetic field that is constant over an encounter region" [Ref. 1:p. 20]. The magnetic signal at a sample point is determined by the Anderson formulation, and its duration is determined by the length of time it takes a magnetometer to pass through a distance that is a function of the slant range between the magnetometer and the magnetic dipole [Ref. 1:p. 24].

B. THE DETECTION MODELS

The encounter model has two alternative binary detection models: a crosscorrelation detection model and a square law detection model. Both models are based on the following assumptions: the magnetic noise is gaussian, the noise samples are independent, and target course and speed remain constant during the detection encounter.

1. Detection Statistics

Let v_1, v_2, ... , v_m represent sample values in a sample interval. Here, a sample value v_i is the voltage generated by a magnetometer at time t_i. In the detection models, a detector uses these sample values to determine a detection statistic x for the sample interval. The decision rule used by both detection models is: If $x \geq x^*$ then the sample interval contains target signal plus noise, otherwise the sample interval contains noise only. Here, x^* is a value that is determined by p_f, the false alarm probability.

For both detection models, if there is no signal present, $v_i = n_i$ where n_i is the noise present at the magnetometer and $i = 1, 2, \ldots, m$. If there is signal present, $v_i = n_i + s_i$ where s_i represents the signal present at the detector. The average signal power $S = (1/m)\bullet\Sigma s_i^2$ and the average noise power $N = \sigma^2$ where σ is the standard deviation associated with the noise process.

2. Crosscorrelation Detection Model

The crosscorrelation detection model is based on the additional assumption that the detection system has knowledge of the statistical characteristics of the noise process and a complete prior knowledge of the input signal. This detection model can provide an upper bound on detection performance. It can be argued that a system that utilizes a computer library of signal characteristics will approach this upper bound.

In this model,

$$x = \Sigma\, v_i \bullet s_i$$

where $i = 1, 2, \ldots, m$ and the sum is over the values corresponding to a sample interval. When there is no signal energy present, x represents a normal random variable with $\mu_X = 0$ and variance $\sigma_X^2 = \sigma^2 \bullet \Sigma\, s_i^2$. The false alarm probability is given by:

$$p_f = 1 - P(x^*/\sigma_X)$$

where P(x) is the standard normal cumulative distribution function. This equation is used to determine the value for x^* in this model.

When the sample interval contains signal energy, x is the value of a normal random variable with $\mu_X = \Sigma\, s_i^2$ where $i = 1, 2, \ldots, m$ and the sum is over the values corresponding to a sample interval. The result is that the probability of detection p_d is given by:

$$p_d = 1 - P(x^*/\sigma_X - \Sigma\, s_i^2/\sigma_X).$$

3. Square Law Detection Model

The square law detection model differs from the crosscorrelation detection model in that the signal is determined by a stochastic process and only the statistical characteristics of the signal process are known. This detection model can provide a lower bound on detection performance.

The statistic x for the square law detector is:

$$x = \Sigma\ v_i^2$$

where $i = 1, 2, \ldots, m$ and the sum is over the values corresponding to the sample interval.

When there is no signal present, x/σ^2 is the value of a chi-square random variable with m degrees of freedom. The false alarm probability is given by:

$$p_f = 1 - P(x^*/\sigma^2 \,|\, m)$$

where $P(x^*/\sigma^2 \,|\, m)$ is the cumulative distribution function for a chi-square random variable with m degrees of freedom. This equation is used to determine the value for x^* in this model.

When the sample interval contains signal energy, x/σ^2 is the value of a noncentral chi-square random variable with m degrees of freedom and a noncentral parameter $\Sigma\, s_i^2/\sigma^2$. The result is that for this model the probability of detection is given by:

$$p_d = 1 - P(x^*/\sigma^2 \,|\, m, \Sigma s_i^2/\sigma^2).$$

A more detailed discussion of the crosscorrelation and the square law detection model is provided in Reference 2.

C. PROGRAM MODIFICATIONS

Based on the lateral range curves that are described in Chapter III, to determine the probability of detecting a target transiting a MAD buoy field, the sweep width for each buoy was calculated. In an isotropic noise field and direct path environment, it is a relatively straight forward procedure to determine a sweep width for a sonobuoy, since the probability of detection for a submarine depends only on its range from the sonobuoy. However, for a MAD buoy on the other hand, the probability of detecting a submarine target for a MAD buoy is dependent on the targets course as shown in Fig. 1.

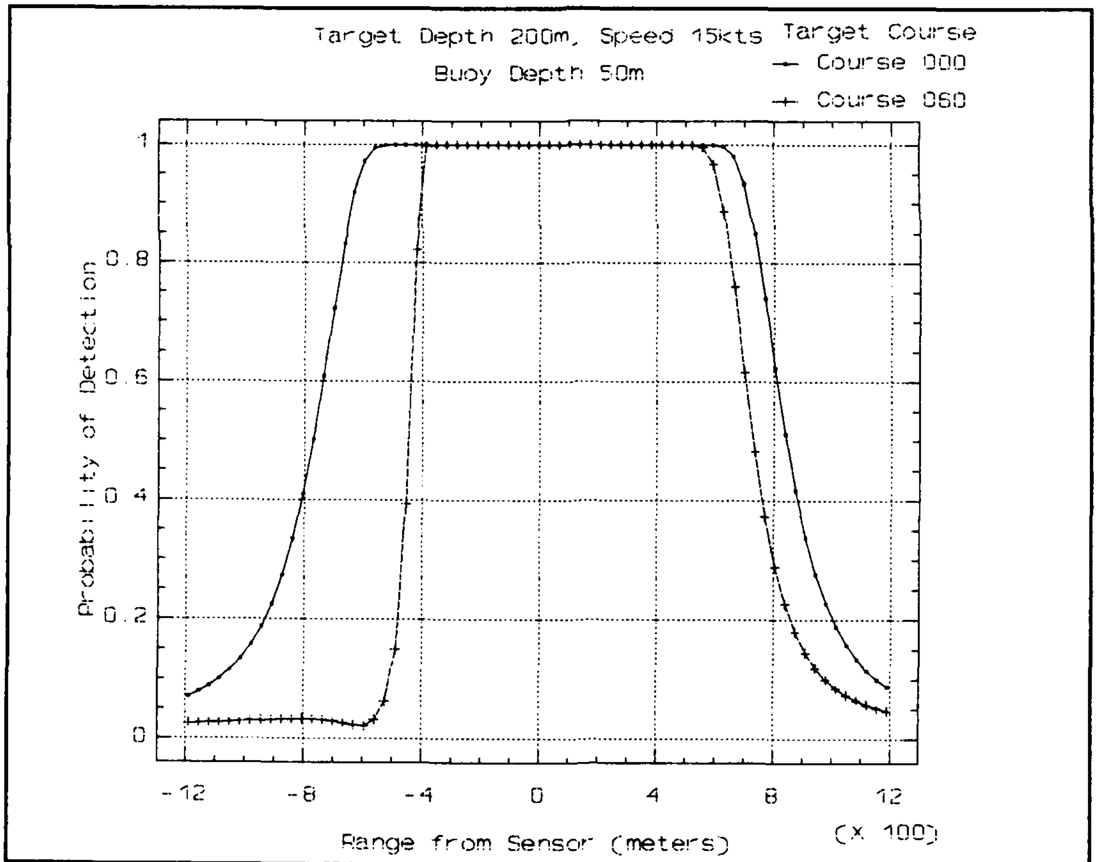

Figure 1. Comparisons of the Lateral Range Curves for a Target at a Constant Depth and Speed with Different Courses

The program to determine lateral range curves that is described in Ref. 1 was modified so that it would determine lateral range curves for encounters involving various target depths and target courses in a single computer run. In addition, the output of the program was altered to provide the sweep width for each target depth and course.

1. Target Depth

In the altered program, a user can input a minimum target depth, maximum target depth, and an interval between calculated encounter depths. The depth inputs allow the user to investigate a broad spectrum of possible encounter depths.

2. Target Course

In addition, in the altered program a target's course varies sequentially from 000 - 150 degrees true by 30 degree increments. It is not necessary for the program to run the reciprocals for the courses indicated above since, for instance, the detection range for 000 degrees true would be the same as 180 degrees true. Figure 2 provides a comparison of the lateral range curve of a target on reciprocal headings. Notice that the two curves are mirror images of each other. For a target course of 180 degrees true, the lateral range curve crosses the .6 probability of detection line at -800 meters whereas the curve for a target course of 000 degrees true crosses the .6 probability of detection threshold at +800 meters.

3. Random Search Model

A random search model described in Reference 3, pp. 18 has been incorporated into the computer simulation to calculate the probability of detection for a MAD buoy field. A random search model was chosen because other search models tend to be overly optimistic in their predictions [Ref 4:p. 2-1].

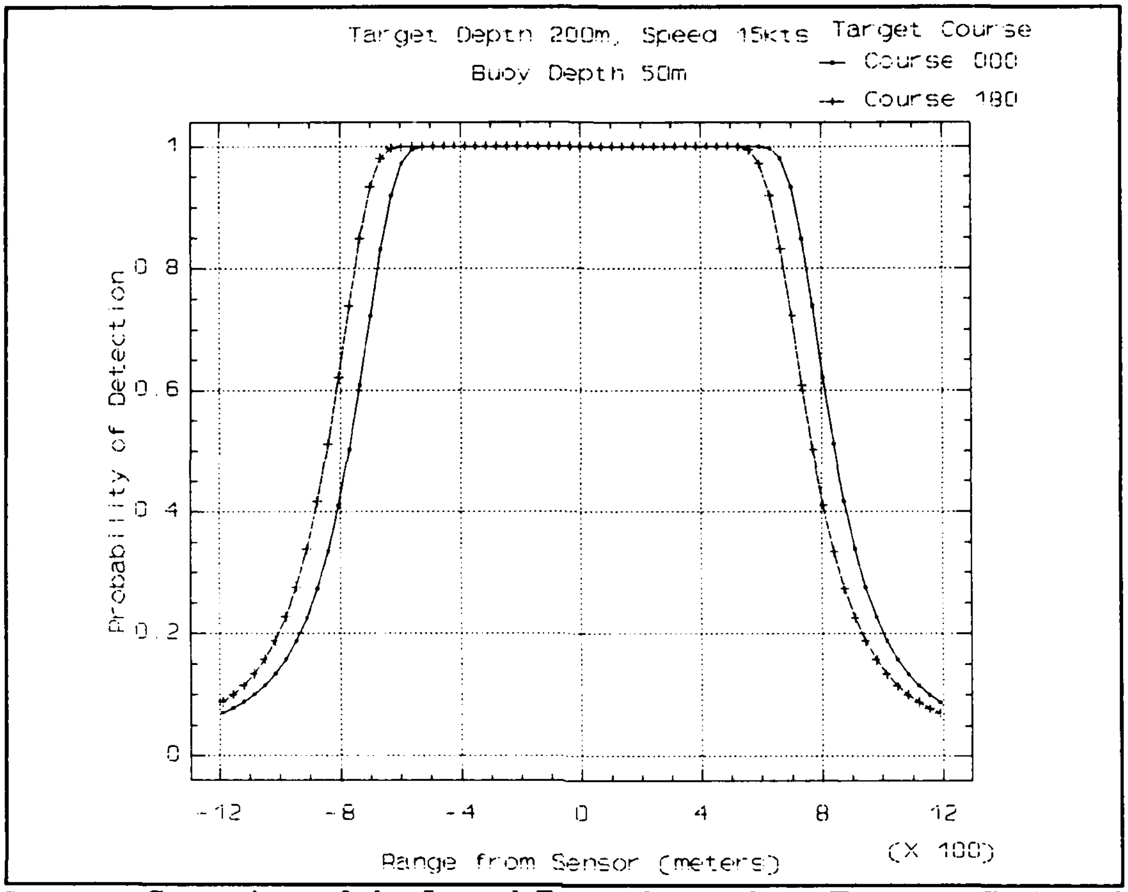

Figure 2. Comparison of the Lateral Range Curves for a Target on Reciprocal Headings.

The search model is based on the assumption that the targets motion is unknown and considered to be random within a defined region of area A. For a buoy field, the probability of detection is:

$$P(det) = 1 - \exp(-nfWu\tau/A)$$

where n is the number of buoys, f is the fraction of buoys that can be monitored simultaneously, W is the sweep width of a buoy, u is the submarine's speed, τ is the duration of the search, and A is the area of the search region [Ref. 3:p. 18].

A necessary condition for this formula to be justified is that the detection regions for the MAD buoys do not significantly overlap. If significant overlap occurs the actual probability of detection could be less than the calculated value.

III. INPUT PARAMETERS

The input parameters for the computer program can either be entered at the keyboard, stored in a single file or divided into three separate files: the magnetic file, the signal processing file, and the kinematic file.

A. MAGNETIC PARAMETERS

1. The Earth Magnetic Field and Dip Angle

Like its prototype, the modified program has the capability of either receiving values of the earth magnetic field H_e and values of the dip angle Φ as direct inputs or of calculating these values using an internal model. Direct input values for the earth magnetic field can be obtained from Defense Mapping Agency Hydrographic Center Chart #39 and values for the dip angle can be obtained from Defense Mapping Agency Center Chart #30.

The values for the dip angle computed by the internal model will, in general, differ from those obtained directly from Chart #30. Table 1, which is taken from Reference 1, shows that a small difference in the computed and chart values for the dip angle does not imply that there exists a small difference between the computed and charted values for the earth magnetic field at the same location. For example, at 60°N and 30°W the difference between the charted dip angle and the calculated dip angle is only 1° while the difference between the charted and calculated earth magnetic field intensity is .12 oersted.

2. The Submarine Magnetic Dipole

The computer program allows the user to input permanent magnetic moments, target displacement, and target permeability factors or coefficients. The program uses these inputs to calculate a submarines dipole moment p. Since the likelihood that the permanent magnetic moments for a submarine contact will be known is small, the values for the permanent magnetic moment are assumed to be zero. This approximates the condition where the submarine has been effectively depermed.

The value for the target displacement used in this thesis is an average of displacements for various Soviet submarine classes. The information used to calculate the average was obtained from *Jane's Fighting Ships 1989-90* [Ref. 5].

Table I. SOME CORRESPONDING VALUES FOR Φ AND H_e.

Lat.	Long.	Program Dip Φ	Chart #30 Dip Φ	Program Field H_e^*	Chart #39 Field H_e
60°N	180°W	74°	70°	.63	.53
60°N	30°W	75°	74°	.64	.52
30°N	90°W	62°	60°	.55	.51
30°N	60°W	59°	59°	.53	.48
30°N	30°E	37°	43°	.41	.42
0°	0°	-48°	-25°	.35	.31
30°S	180°W	-45°	-55°	.44	.50
30°S	90°W	-30°	-34°	.39	.31
30°S	90°E	-62°	-66°	.55	.52
60°S	30°W	-69°	-56°	.60	.34

* in oersted, 1 oersted = 10^5 gamma

The use of this reference is not intended as a endorsement of the accuracy of the reference. It was used only in order to provide a convenient point of departure for the analysis.

The average value $\hat{d}$ for the target displacement was calculated by summing the displacement d for a submarine of class i multiplied by the number n of submarines in class i for all i and dividing this sum by the total number of operational submarines in the Soviet inventory. Using the relation:

$$\hat{d} = \frac{\Sigma(n_i \bullet d_i)}{\Sigma n_i}$$

the average value of a submarine contact was 5600 tons.

The final piece of information needed is the permeability factors or the permeability coefficients. The permeability coefficients k_i are related to the submarine displacement d and the permeability factor f_i by the equation:

$$k_i = f_i \bullet d ,$$

where i is the index for transverse, longitudinal, and vertical axis [Ref. 1:p. 34]. The computer program will accept either permeability coefficients or permeability factors as inputs.

In this thesis, the permeability factors were used as an input for the program. The input values were obtained from Texas Instrument Report C2-61009-2 [Ref. 6:p. 4] and are based on a WW II vintage submarine. Care must be exercised when using these inputs because the units for the Texas Instrument inputs are foot^3/ton. The program requires inputs with units of oersted-centimeter^3/gamma-ton. This problem is easily solved by dividing the Texas Instrument inputs by 3.53 [Ref. 1:p.35].

B. PROCESSING PARAMETERS

The processing parameters are crucial in that these parameters determine the type of magnetometer being modeled. The computer program is flexible in that it will accept direct inputs for the sample interval and the sample rate or it will calculate these inputs if they are not known beforehand.

1. Sampling Rate

The program uses the value 2 • MAXF as the sampling rate where MAXF is defined to be the maximum magnetic signal frequency. This is the Nyquist rate for an ideal low pass filter [Ref. 1:p. 9]. In the program, MAXF is 2•MAXVM/MINRO where MAXVM is the maximum encounter relative speed in meters per second and MINRO is the minimum slant range in meters of a just detectable target. A just detectable target is defined to be the smallest target in terms of its magnetic dipole moment that can be detected with an acceptable probability. Since the magnetometer on a buoy is stationary, MAXVM equals the maximum speed of the target in knots converted to meters per second. Additional discussion of the sampling rate can be found in [Ref. 7:pp. 29-30].

2. Integration Time

The program calculates the sample interval length (integration time) as 2•MAXRO/MINVM where MINVM is the estimated minimum encounter relative speed in knots converted to meters per second and MAXRO is the estimated maximum slant range at CPA for an encounter for the maximum expected dipole moment to be encountered. Additional discussion of the integration time can be found in [Ref. 7:pp. 28-29].

3. Processing Inputs

The hypothetical MAD detection system modeled in this thesis has a maximum estimated slant range at CPA of 155 meters for a just detectable target. The maximum slant range at CPA for a target with the largest expected dipole moment is estimated to be 610 meters. The minimum expected encounter speed is 5 kts and the maximum expected encounter speed is 10 kts.

C. KINEMATIC PARAMETERS

The kinematic inputs include the values for magnetometer course, depth, and speed as well as the target course, depth, and speed. The kinematic inputs are basically self explanatory and with the exception of target depth, require no explanation.

There are three depth parameter inputs: minimum target depth, maximum target depth, and depth interval. The minimum and maximum target depths define the range of values for which the simulation will calculate the probabilities of detection. The default inputs are 0 meters and 700 meters. The maximum target depth of 700 meters was arbitrarily chosen by using Ref. 5 to determine the maximum service depth of current Soviet submarines. The use of 700 meters as the maximum service depth does not constitute a statement confirming the accuracy of the source.

The default value for the depth interval is 50 meters. This value was determined to be an appropriate compromise between the longer run time that would result from a smaller interval and the need to provide an adequate depth resolution.

III. ANALYSIS

The affects of false alarm rate, noise, buoy depth, and submarine displacement on detection are discussed in this chapter.

A. FALSE ALARM RATE

The simulation's crosscorrelation and square law detection models use the input false alarm rate to determine the probability of false alarm p_f. This value represents the probability that a "signal present" decision will be made when in fact there is no signal present. As $\Sigma\ s_i^2$ approaches zero p_d approaches p_f. This relationship can be seen in Figure 3, at ranges greater than 1500 meters, the lateral range curves are converging to p_f. All of the parameters for the two curves shown in Figure 3 are identical with the exception of the false alarm rate. In this scenario the p_f generated by a false alarm rate of 3 false alarms per hour is .395 and the p_f generated by a false alarm rate of .1 false alarms per hour is .013. Figure 3 illustrates the point that the lateral range curve with a large p_f dominates the lateral range of a smaller p_f.

For an airborne MAD detector, a false alarm rate of .3 false alarms per hour can be considered conservative however, for a MAD buoy field, a false alarm rate of 3 false alarms per hour could be considered excessive while .1 false alarms per hour could be considered conservative. The trade off is that while a larger p_f results in a larger p_d, the higher false alarm rate leads to an increased number of false contacts that have to be resolved. To put this point in perspective, consider a MAD buoy field that consists of 10 buoys with each buoy having a life of 10 hours. A false alarm rate of 3 false alarms per hour for each buoy would result in, on the average,

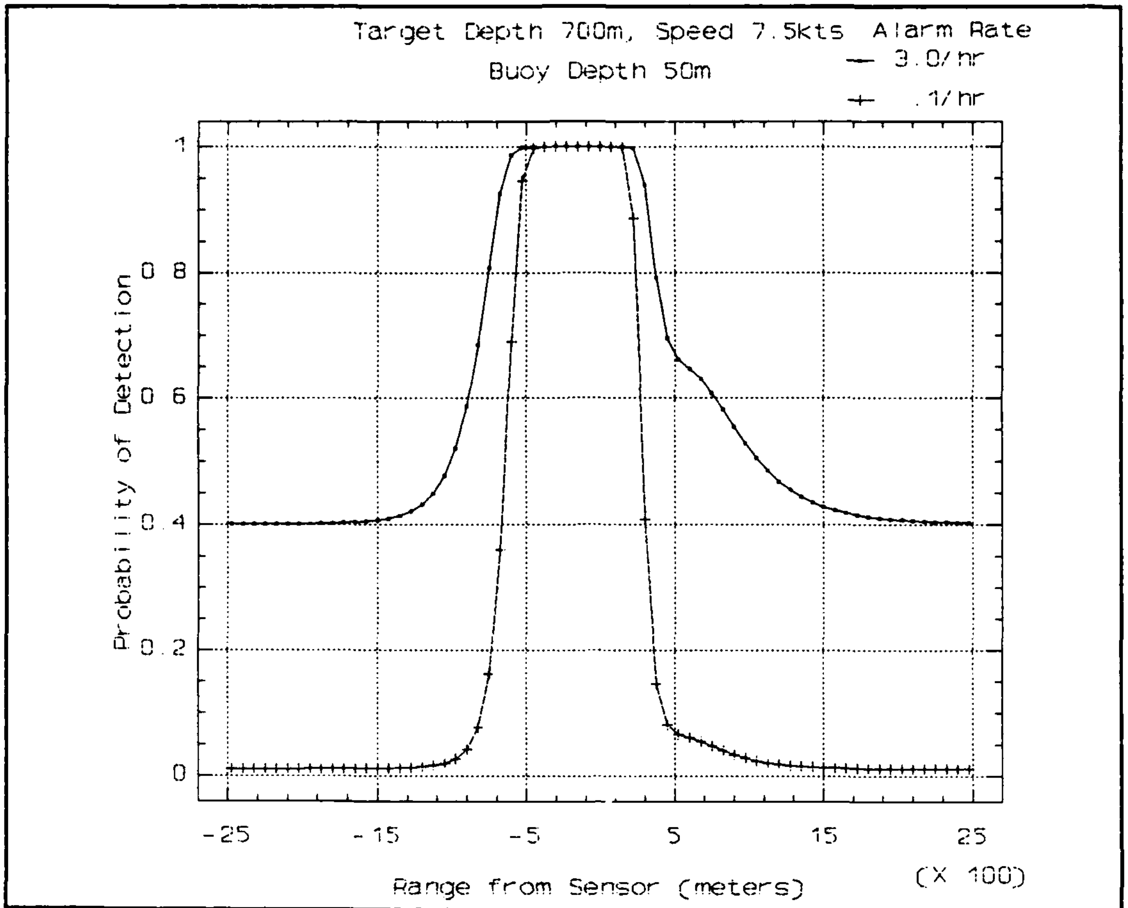

Figure 3. Comparison of the Lateral Range Curves for Two Different False Alarm Rates.

30 false alarms per hour for the buoy field. If each buoy is assumed to have a 10 hour life then this would result in, on the average, 300 false alarms in 10 hours. On the other hand, a false alarm rate of .1 false alarms per hour for each buoy would, on the average, result in 1 false alarm per hour or 10 false alarms in 10 hours.

B. SWEEP WIDTH

The sweep width W of a detection system is the area under the lateral range curve and it is formally defined by:

$$W = \int_{-\infty}^{\infty} p(x)dx$$

where x is the lateral range and p(x) is the probability of detecting the target at a closest point of approach (CPA) range x during a complete straight line encounter. For a definite range law detection system with definite range R, the sweep width is $W = 2 \bullet R$.

It is readily apparent that the sweep width calculated using the formal definition given above for the curves in Figure 3 would result in an infinitely large sweep width in either case. This suggests that a definite range law formula with R equal to the average of the .5 probability of detection ranges would be appropriate to define the sweep width. However, using the definite range law formula would conceal the asymmetry found in the magnetometer lateral range curve. For false alarm rates resulting in a p_f of .1 or less, a value for the sweep width equal to the an area under the lateral range curve was used where the limits of integration were the ranges at which p_d equals a value significantly close to p_f. This is a better choice than that above, since it can provide a more realistic representation of the detection process. For the analysis conducted in this thesis, the false alarm rate was arbitrarily set at .1 false alarms per hour which results in a p_f of .013 and the limits of integration were from the negative range to the positive range where p_d equaled .015. Figure 4 illustrates, for a square law model, the difference in the sweep width determined by the two methods of calculation. Since the lateral range curve for a buoy varies as a function of target course, a representative sweep width for the random search model was required. Based on its assumption of a uniform target distribution, a value was found by averaging the sweep widths that were determined for the target courses 000, 020, 060, 090, 120, and 150 degrees true.

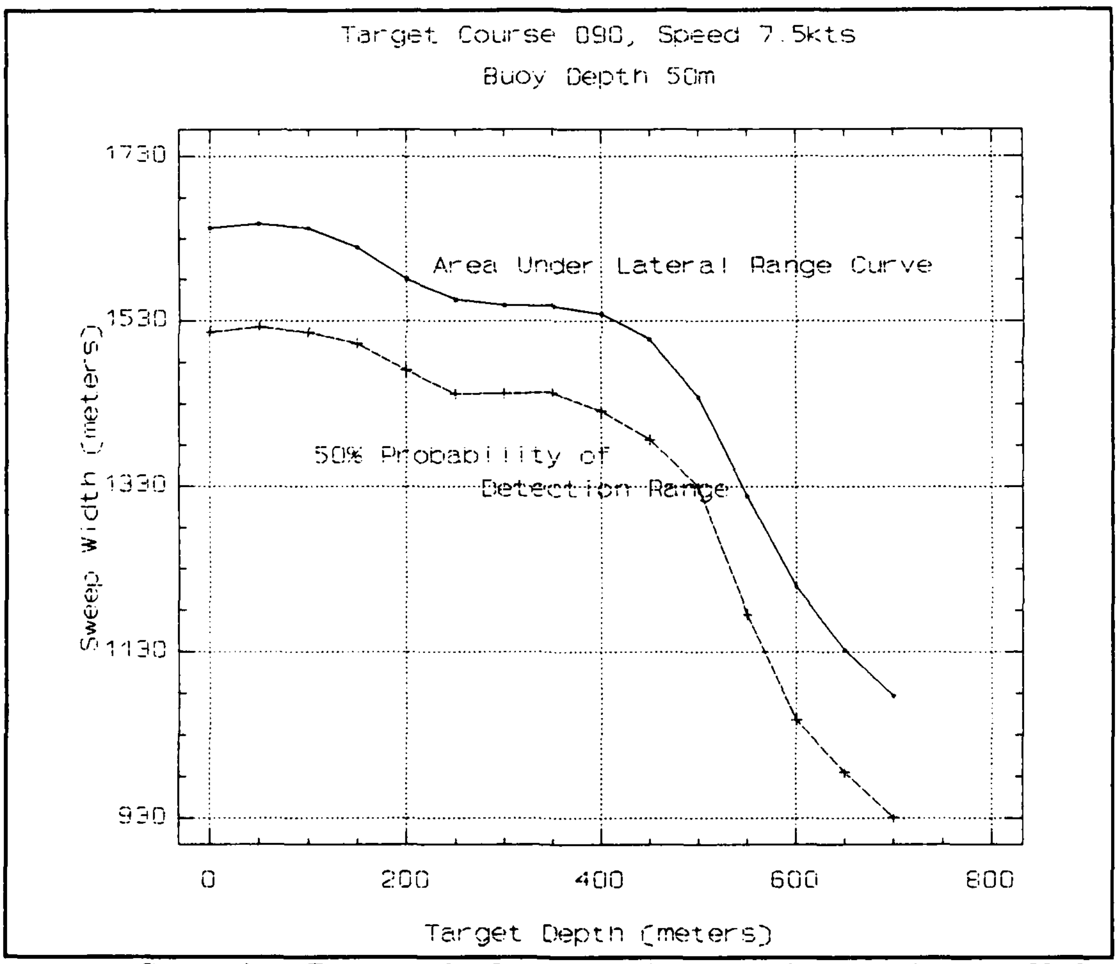

Figure 4. Comparison Between the Sweep Width as Calculated as the Area Under the Lateral Range Curve vs. the 50% Probability of Detection Range.

C. RANDOM SEARCH MODEL SENSITIVITY ANALYSIS

This section describes the results of a sensitivity analysis of the random search model. The parameters noise level, buoy depth, and target displacement were individually varied in the analysis for a buoy field consisting of 10 MAD buoys randomly placed within a 15 nm by 15 nm square. The search duration is 2 hrs and the target speed is 7.5kts. The fraction of the MAD buoys that can be monitored at one time is assumed to be 1.

1. Noise Level

Four different values for the magnetic noise were used in this analysis: 10 gamma, 1 gamma, .1 gamma, and .01 gamma. As expected, an increase in magnetic noise lead to a significant decrease in the detection range for the buoy field, as shown in Figure 5. What is interesting is that the shape of the curves as a function of depth change as the noise levels change.

Notice that if the magnetic noise is lowered to .01 gamma, there is not much variation between the buoy field probability of detections as a function of depth to a depth of 800 meters. This occurs because to a depth of 800 meters, the signal-to-noise ratio remains large and, consequently, so does the sweep width. As the magnetic noise increases, the variation between buoy field probability of detections increases due the decreasing signal-to-noise ratio. If the magnetic noise is increased to 1 gamma, at depths of greater than 500 meters the curve begins to flatten out due to the convergence of p_d with p_f. This effect becomes even more noticeable when the noise is increased to 10 gamma. At depths greater than 250 meters the buoy field detection probabilities become constant. The magnetic target signal has decreased to the point were, at its maximum value, p_d is only slightly larger than p_f, resulting in a lateral range curve that is relatively flat.

2. Buoy Depth

The buoy field probabilities of detection were calculated for three different buoy depths: 50 meters, 100 meters, 150 meters. Table II contains the field probabilities of detection for the three buoy depths. Notice that the maximum field detection probability occurs at the depth of the buoy and that this maximum value remains constant over the three depths. This result is consistent with the fact that, because of the magnetic signal model, only the vertical distance between the target and the buoy is required to determine the sweep width.

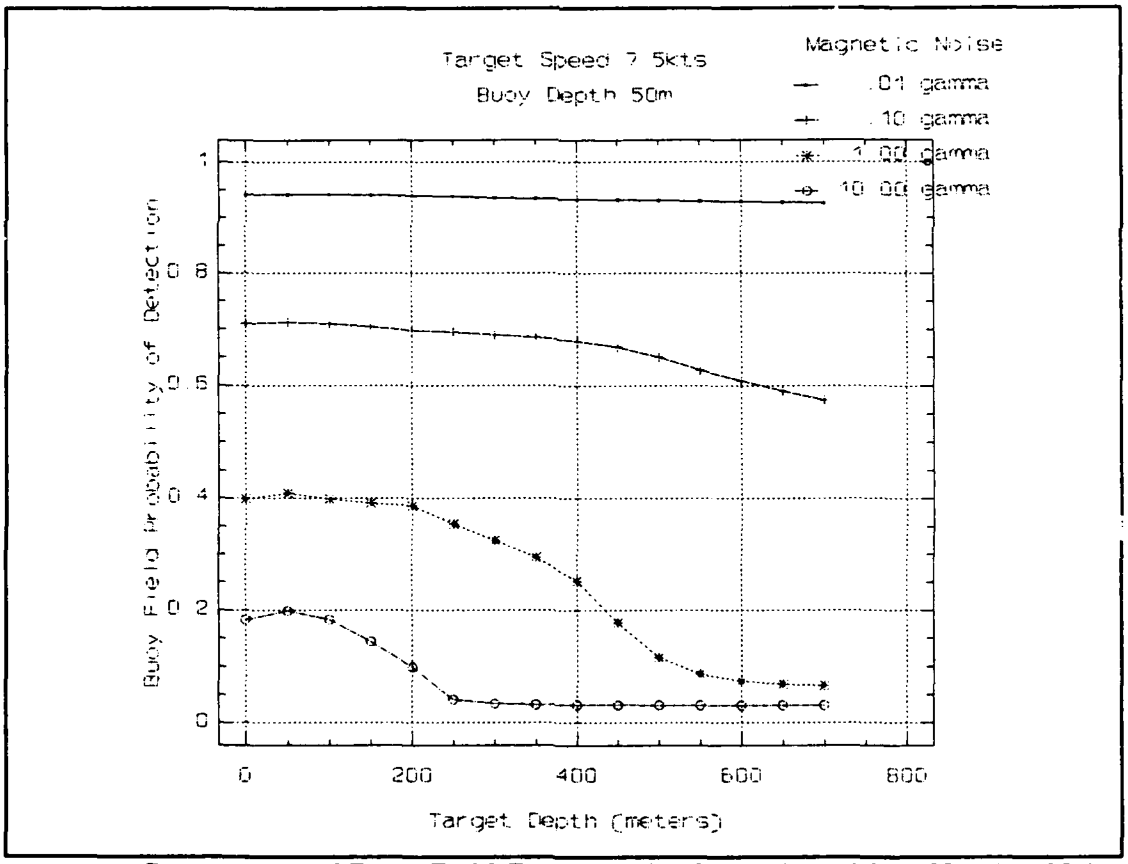

Figure 5. Comparison of Buoy Field Detection Probabilities with Varying Noise Values Using the Square Law Detection Model.

The final computer simulation program shown in Appendix A utilizes this principle to provide buoy field detection probabilities for a range of buoy depths, in one simulation run, without significantly increasing the simulation runtime. For example in Table II, if the field detection probabilities are known for the maximum buoy depth of 150 meters (Column 3) the field detection probabilities for all buoy depths less than the maximum but equal to the depth sample interval of 50 meters (Columns 1 and 2) are also known.

3. Target Displacement

This section describes a sensitivity analysis of the effects of a change in displacement. The maximum and minimum displacements studied were 22,400 tons and 700 tons.

Table II. TABULATION CF MAD BUOY FIELD DETECTION PROBABILITIES FOR VARYING TARGET AND BUOY DEPTHS.

Target Depth	Buoy Depth		
meters	50 meters	100 meters	150 meters
0	.710	.704	.697
50	.712	.710	.704
100	.710	.712	.710
150	.704	.710	.712
200	.697	.704	.710
250	.695	.697	.704
300	.690	.695	.697
350	.685	.690	.695
400	.678	.685	.690
450	.668	.678	.685
500	.650	.668	.678
550	.628	.650	.668
600	.608	.628	.650
650	.591	.608	.628
700	.574	.591	.608
750	-	.574	.591
800	-	-	.574

Figure 6 shows buoy field detection probabilities for a square law detector. Notice that, over the range of operational target depths, the buoy field probability of detection curves for smaller displacements experience a sharper drop off in detection probability than the curves for larger target displacements. If target depths significantly greater than 700 meters had been displayed, then the same sharp decrease in the buoy field detection probability curves would have been observed for the larger displacements.

Table III provides various statistical details about the sweep width for the different target displacements averaged over depths. Target depth was assumed to

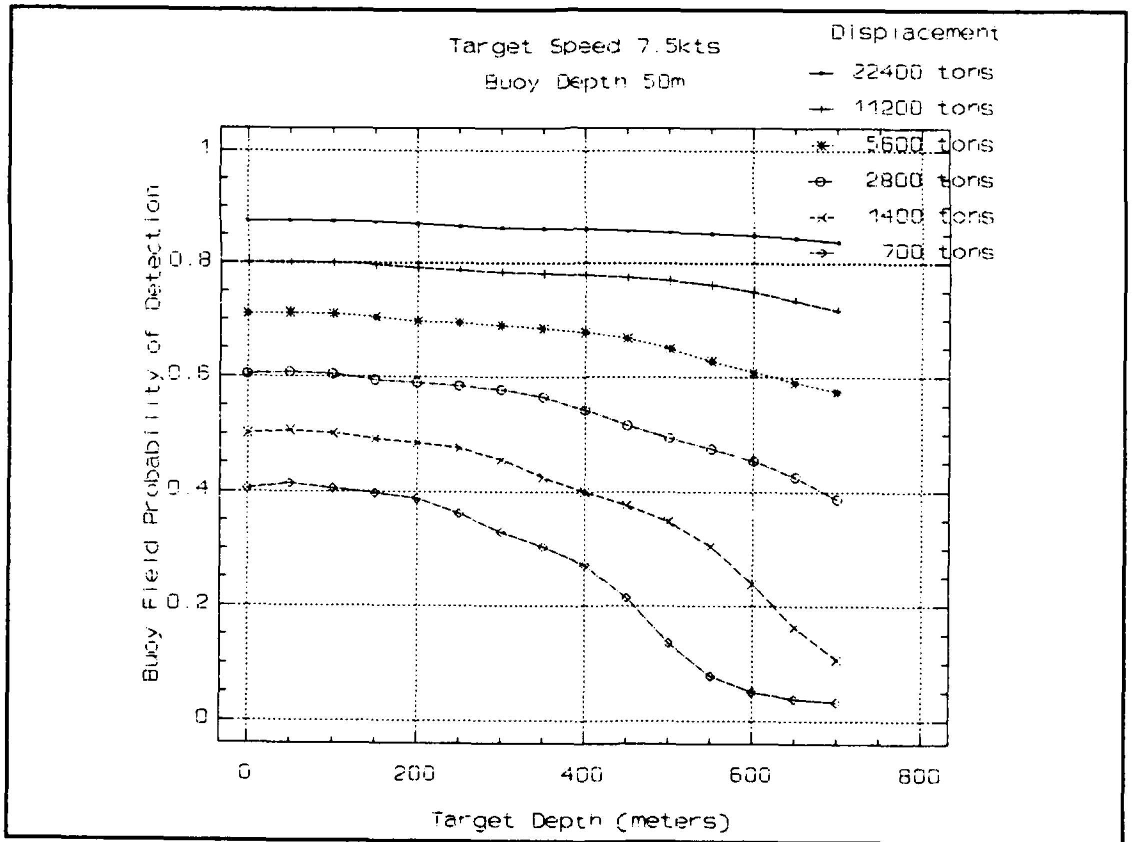

Figure 6. Comparison of Buoy Field Detection Probabilities with Varying Target Displacements and Magnetic Noise of .1 Gamma.

Table III. STATISTICAL ANALYSIS OF THE AVERAGE SWEEP WIDTH FOR VARIOUS DISPLACEMENTS

	Target Displacement in tons					
	22,400	11,200	5,600	2,800	1,400	700
Sample Size	15	15	15	15	15	15
Average	2820*	2137	1578	1106	720	446
Median	2817	2169	1645	1185	788	516
Mode	2804	2298	1765	1113	726	742
Standard Deviation	117	154	188	212	280	280
Minimum	2560	1796	1218	699	161	47
Maximum	2969	2305	1774	1333	1006	763
Interquartile Range	216	230	327	370	444	606

*All sweep widths are in meters

be uniformly distributed between 0 meters and 700 meters. As expected the average of the sweep width over all depths decreases as the target displacement decreases. However, the range of the values over all depths increases from a minimum of 370 meters at a target displacement of 22,400 tons to a maximum of 845m at a target displacement of 1400 tons. For a target displacement of 700 tons the range of values begins to decrease again due to the small magnetic dipole moment which results in the sweep width converging to a minimum value. The standard deviation follows the same pattern, it is inversely proportional to the target displacement. The larger the displacement the smaller the standard deviation.

This relationship can be seen in Figure 6. The buoy field detection curve for a 22,400 ton target is relatively flat, while the curve for a 1400 ton target exhibits a more pronounced curve. The detection curve for a 700 ton target actually has an inflection point at about 500 meters target depth. This inflection point is caused by the fact that the average sweep width is converging to a value greater than zero.

Analysis in this section suggests that, for small submarine displacements, it becomes more important to place a buoy's magnetometer near the target depth to ensure an even marginal probability of detection under high noise conditions. For example, in Figure 6, the rate of change for a 22,400 ton target (the upper curve) is relatively small over the operational target depths. In this case placing the buoy magnetometer at a depth other than the target depth does not result in a significant loss of detection probability. Conversely the curve for a 1400 ton target displays a significant rate of change and the magnetometer's depth has a large effect on the ability of the buoy field to detect the target. Section four will explore the effects of placing the buoy magnetometers at a variety of depths within the same field.

4. Multiple Buoy Depths

In this section, the effect that placing the buoy magnetometers within the same field at various depths has on the detection probabilities curves is discussed. For this analysis the target displacement was assumed to be 1400 tons and the 10 buoy magnetometers were placed at three different depths: three at 100 meters, four at 350 meters, and three at 600 meters. The size of the search area remained the same as in previous analysis.

The buoy field detection probabilities were calculated using the following formula:

$$p(\text{det}) = \Sigma\ c_i \bullet p_i$$

where c_i is the fraction of the buoys at depth i and p_i is the probability of detecting the target at depth i with all the buoys in the field. The fraction of the buoys at the three assigned depths were as follows: .3 at 100 meters, .4 at 350 meters, and .3 at 600 meters. Figure 7 illustrates the results of the simulation.

It was anticipated that placing the buoy magnetometers at a variety of depths within the same field would increase the probability that the buoy field would detect the target. It is apparent from Figure 7 that, contrary to the expectations, the buoy field detection probability curve for a field depth of 350 meters dominates the multiple depth buoy field detection probability curve for all depths under consideration. This suggests that, assuming a uniform target depth distribution, deploying the buoy magnetometers to the average target depth would

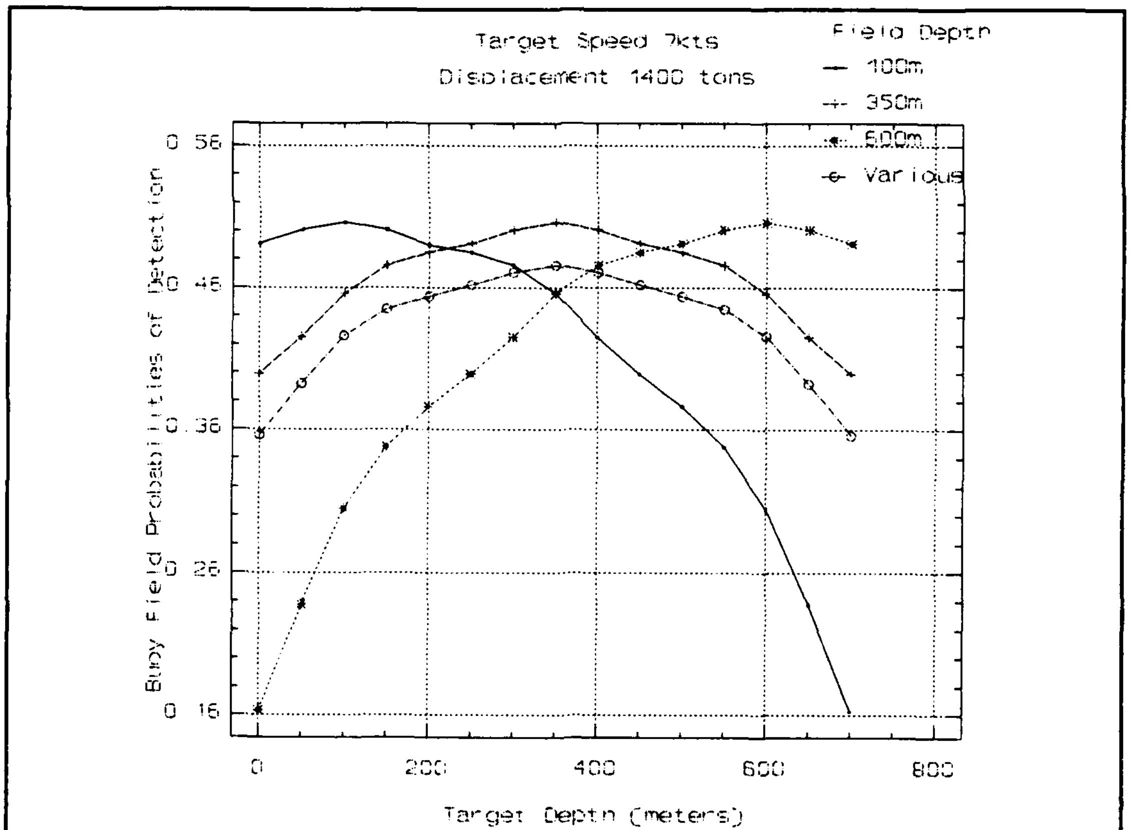

Figure 7. Comparison of Buoy Field Detection Probability Curves for Sensor Depths of 100m, 350m, 600m, and Various Sensor Depths With In the Same Buoy Field.

result in the best probability of detecting the target. In order to confirm the results, a number of different scenarios were run varying buoy depth separation and target displacement and Figure 7 is representative of the results.

IV. CONCLUSIONS

This thesis presented an analysis of a Magnetic Anomaly Detection buoy field that is based on data generated with a computer simulation. The simulation uses a crosscorrelation detection system model to provide an upper bound and a square law detection system model to provide a lower bound on the detection system performance. A random search model was used to provide a lower bound on the search effectiveness of the buoy field.

Analysis has shown that the choice of the false alarm rate is the deciding factor in choosing the method used to calculate the sweep width. False alarm rates that result in a p_f greater than .1 suggests that a definite range law model be used. For a p_f that is less than or equal to .1 this assumption is not necessary since p_f is sufficiently close to zero so that limits of integration for estimating the area under the lateral range can be chosen without inducing a large error. This method should provide a more realistic representation of the detection process.

The random search model sensitivity analysis for noise level, buoy depth, and target displacement revealed that the variation between the buoy field detection probability as a function of depth was greater for the higher noise levels. The lowest noise levels showed relatively little variation in the buoy field detection probability over the assumed submarine operational depths, while the highest noise levels had a significant increase in the variation of the buoy field detection probability. The variability of the buoy field detection probability as a function of depth was decreased as the submarine displacement increased. The buoy field detection probability was maximized when the buoy magnetometer was at the same depth as the target. This indicates that the depth placement of the buoy magnetometer becomes more important as noise increases or as submarine displacement decreases. As expected, varying the buoy depth also confirmed that the vertical separation between the buoy magnetometer and the target is the key factor and not the buoy depth. Target displacement analysis exhibited the same general trend as the noise level analysis. The rate of change for the large displacement targets show a less significant rate of change than the smaller displacement targets. Finally, analysis showed that varying the buoy magnetometer depth within the same buoy field does not improve the buoy field search effectiveness over a buoy field with all the buoy magnetometers at the average target depth for a uniform target depth distribution.

APPENDIX A. A PROGRAM LISTING

```
10 CLS : CLEAR
20 PRINT "Magnetic Anomaly Detection (MAD) Buoy Field Simulation Program"
30 'REM This simulation is based on a program to compute the Magnetic
40 'REM Anomaly Detection (MAD) lateral range function, developed by
50 'REM Prof. R. N. Forrest, Naval Postgraduate School, Monterey, CA.
60 DIM X0(-1 TO 70), R0(70), H(70), HS(70, 150), K(70), PDCC(71), PDSL(71),
AREA1T(70), AREA2T(70)
70 PI = 3.141592654#: CON = 2 * PI / 360: KON = 1852 / 3600: N$ =
"MAD.BAS"
80 Q0 = .2316419: Q1 = .31938153#: Q2 = -.356563782#: Q3 = 1.781477937#
90 Q4 = -1.821255978#: Q5 = 1.330274429#
100 I1 = 2.515517: I2 = .802853: I3 = .010328: I4 = 1.432788: I5 = .189269: I6 =
.001308
110 PRINT
120 INPUT "generate data or print a program data file (g/p)"; A$
130 IF A$ = "G" OR A$ = "g" THEN GOTO 150
140 IF A$ = "P" OR A$ = "p" THEN GOTO 3540 ELSE GOTO 120
150 PRINT : A$ = "a"
160 INPUT "magnetic, processing & kinematic data entry by combined file (y/n)";
A$
170 IF A$ = "Y" OR A$ = "y" THEN GOTO 1950
180 IF A$ = "N" OR A$ = "n" THEN GOTO 190 ELSE GOTO 160
190 INPUT "magnetic data entry by file/keyboard (f/k)"; A$
200 IF A$ = "F" OR A$ = "f" THEN GOTO 810
210 IF A$ = "K" OR A$ = "k" THEN GOTO 220 ELSE GOTO 190
220 INPUT "latitude in decimal degrees (N +)"; LAT: LATR = LAT * CON
230 INPUT "longitude in decimal degrees (E +)"; LNG: LNGR = -LNG * CON
240 INPUT "variation in decimal degrees (E +)"; DEC: DECR = DEC * CON
250 A$ = "a"
260 INPUT "input dip angle (y/n)"; A$
270 IF A$ = "Y" OR A$ = "y" THEN GOTO 290
280 IF A$ = "N" OR A$ = "n" THEN GOTO 320 ELSE GOTO 260
290 INPUT "dip angle in decimal degrees (north magnetic hemisphere +)"; DIP
300 DIPR = DIP * CON
310 GOTO 420
320 LATPR = 76 * CON: LNGPR = 100 * CON
330 D = SIN(LNGR - LNGPR) * COS(LATR): E = COS(LNGR - LNGPR) *
COS(LATR): F = SIN(LATR)
340 X = E: Y = F
350 GOSUB 4430
360 T = T - (90 * CON - LATPR): E = R * SIN(T): F = R * COS(T)
370 X = E: Y = D
380 GOSUB 4430
```

```
390 X = F: Y = R
400 GOSUB 4430
410 DIPR = ATN(2 * TAN(T)): DIP = DIPR / CON
420 A$ = "a"
430 INPUT "input encounter magnetic field intensity (y/n)"; A$
440 IF A$ = "Y" OR A$ = "y" THEN GOTO 460
450 IF A$ = "N" OR A$ = "n" THEN GOTO 480 ELSE GOTO 430
460 INPUT "encounter magnetic field intensity in gamma"; HE
470 GOTO 490
480 HE = 70000! / SQR(3 * COS(DIPR) * COS(DIPR) + 1)
490 PLM = 0: PTM = 0: PVM = 0
500 A$ = "a"
510 INPUT "input target permanent dipole moments (y/n)"; A$
520 IF A$ = "Y" OR A$ = "y" THEN GOTO 540
530 IF A$ = "N" OR A$ = "n" THEN GOTO 570 ELSE GOTO 510
540 INPUT "permanent longitudinal moment in oersted-cm3 (stern-to-bow +)"; PLM
550 INPUT "permanent transverse moment in oersted-cm3 (port-to-starboard +)";
PTM
560 INPUT "permanent vertical moment in oersted-cm3 (downward +)"; PVM
570 INPUT "target displacement in tons"; WT
580 A$ = "a"
590 INPUT "input target permeability coefficients or factors (c/f)"; A$
600 IF A$ = "C" OR A$ = "c" THEN GOTO 620
610 IF A$ = "F" OR A$ = "f" THEN GOTO 660 ELSE GOTO 590
620 INPUT "longitudinal permeability coefficient in cgs units"; KL
630 INPUT "transverse permeability coefficient in cgs units"; KT
640 INPUT "vertical permeability coefficient in cgs units"; KV
650 FL = KL / WT: FT = KT / WT: FV = KV / WT: GOTO 710
660 INPUT "longitudinal displacement factor in cgs units"; FL
670 INPUT "transverse displacement factor in cgs units"; FT
680 INPUT "vertical displacement factor in cgs units"; FV
690 KL = FL * WT: KT = FT * WT: KV = FV * WT
700 A$ = "a"
710 INPUT "generate a magnetic data file (y/n)"; A$
720 IF A$ = "Y" OR A$ = "y" THEN GOTO 740
730 IF A$ = "N" OR A$ = "n" THEN GOTO 880 ELSE GOTO 710
740 INPUT "magnetic data file name"; M$
750 ON ERROR GOTO 760: GOTO 770
760 RESUME 710
770 OPEN "O", #1, M$
780 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL,
FT, FV
790 CLOSE
800 GOTO 880
810 INPUT "magnetic data file name"; M$
820 ON ERROR GOTO 830: GOTO 840
830 RESUME 190
840 OPEN "I", #1, M$
850 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL,
FT, FV
```

```
860 CLOSE
870 DECR = DEC * CON: DIPR = DIP * CON
880 A$ = "a"
890 INPUT "processing data entry by file/keyboard (f/k)"; A$
900 IF A$ = "F" OR A$ = "f" THEN GOTO 1480
910 IF A$ = "K" OR A$ = "k" THEN GOTO 920 ELSE GOTO 890
920 A$ = "a"
930 INPUT "input sampling period (y/n)"; A$
940 IF A$ = "Y" OR A$ = "y" THEN GOTO 960
950 IF A$ = "N" OR A$ = "n" THEN GOTO 990 ELSE GOTO 930
960 INPUT "sampling period in seconds"; DT
970 IF DT <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT :
GOTO 960
980 GOTO 1120
990 A$ = "a"
1000 INPUT "input maximum magnetic signal frequency (y/n)"; A$
1010 IF A$ = "Y" OR A$ = "y" THEN GOTO 1030
1020 IF A$ = "N" OR A$ = "n" THEN GOTO 1060 ELSE GOTO 1000
1030 INPUT "maximum magnetic signal frequency in Hertz"; MAXF
1040 IF MAXF <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT
: GOTO 1030
1050 GOTO 1110
1060 INPUT "minimum target slant range at CPA in meters"; MINR0
1070 IF MINR0 <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT
: GOTO 1060
1080 INPUT "maximum magnetometer relative speed in knots"; MAXVMK
1090 MAXVM = MAXVMK * KON: MAXF = 2 * MAXVM / MINR0
1100 IF MAXVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT
: GOTO 1080
1110 DT = 1 / (2 * MAXF): REM low pass filter, Nyquist sampling rate
1120 A$ = "a"
1130 INPUT "input integration time (y/n)"; A$
1140 IF A$ = "Y" OR A$ = "y" THEN GOTO 1160
1150 IF A$ = "N" OR A$ = "n" THEN GOTO 1240 ELSE GOTO 1130
1160 INPUT "integration time in seconds"; IT
1170 IF IT >= DT THEN GOTO 1200
1180 PRINT : PRINT "IT = " + STR$(IT) + " seconds      minimum = " +
STR$(DT) + " seconds": PRINT
1190 GOTO 1160
1200 NS = 2 * INT(IT / DT / 2) + 1: REM adj number of samples per integration
time
1210 IF NS <= 151 THEN GOTO 1370
1220 PRINT : PRINT "IT = " + STR$(IT) + " seconds      maximum = " +
STR$(150 * DT) + " seconds": PRINT
1230 GOTO 1160
1240 INPUT "maximum target slant range at CPA in meters"; MAXR0
1250 IF MAXR0 <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT
: GOTO 1240
1260 INPUT "minimum magnetometer relative speed in knots"; MINVMK
1270 MINVM = MINVMK * KON
```

```
1280 IF MINVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT
: GOTO 1260
1290 IT = 2 * MAXR0 / MINVM
1300 IF IT >= DT THEN GOTO 1330
1310 PRINT : PRINT "IT = " + STR$(IT) + " seconds      minimum = " +
STR$(DT) + " seconds": PRINT
1320 GOTO 1130
1330 NS = 2 * INT(IT / DT / 2) + 1: REM adjusted number of samples per
integration time
1340 IF NS <= 151 THEN GOTO 1370
1350 PRINT : PRINT "IT = " + STR$(IT) + " seconds      minimum = " +
STR$(150 * DT) + " seconds": PRINT
1360 GOTO 1130
1370 A$ = "a"
1380 INPUT "generate a processing data file (y/n)"; A$
1390 IF A$ = "Y" OR A$ = "y" THEN GOTO 1410
1400 IF A$ = "N" OR A$ = "n" THEN GOTO 1530 ELSE GOTO 1380
1410 INPUT "processing data file name"; P$
1420 ON ERROR GOTO 1430: GOTO 1440
1430 RESUME 1380
1440 OPEN "O", #1, P$
1450 WRITE #1, DT, IT, NS
1460 CLOSE
1470 GOTO 1540
1480 INPUT "processing data file name"; P$
1490 ON ERROR GOTO 1500: GOTO 1510
1500 RESUME 890
1510 OPEN "I", #1, P$
1520 INPUT #1, DT, IT, NS
1530 CLOSE
1540 A$ = "a"
1550 INPUT "kinematic data entry by file/keyboard (f/k)"; A$
1560 IF A$ = "F" OR A$ = "f" THEN GOTO 1770
1570 IF A$ = "K" OR A$ = "k" THEN GOTO 1580 ELSE GOTO 1550
1580 CM = 0: REM Magnetometer Course(CM) = 0 for a buoy
1590 VMK = 0: REM Magnetometer Speed(VMK) = 0 for a buoy
1600 INPUT "magnetometer depth in meters "; AM: AM = -AM
1610 CT = 150
1620 INPUT "target speed in knots"; VTK
1630 INPUT "maximum target depth in meters"; AT
1640 INPUT "minimum target depth in meters"; ATMIN
1650 INPUT "depth interval in meters"; ATINT
1660 A$ = "a"
1670 INPUT "generate a kinematic data file (y/n)"; A$
1680 IF A$ = "Y" OR A$ = "y" THEN GOTO 1700
1690 IF A$ = "N" OR A$ = "n" THEN GOTO 1840 ELSE GOTO 1670
1700 INPUT "kinematic data file name"; K$
1710 ON ERROR GOTO 1720: GOTO 1730
1720 RESUME 1670
1730 OPEN "O", #1, K$
```

```
1740 WRITE #1, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT
1750 CLOSE
1760 GOTO 1830
1770 INPUT "kinematic data file name"; K$
1780 ON ERROR GOTO 1790: GOTO 1800
1790 RESUME 1770
1800 OPEN "I", #1, K$
1810 INPUT #1, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT
1820 CLOSE
1830 A$ = "a"
1840 INPUT "generate a combined magnetic, processing & kinematic data file (y/n)";
A$
1850 IF A$ = "Y" OR A$ = "y" THEN GOTO 1870
1860 IF A$ = "N" OR A$ = "n" THEN GOTO 2030 ELSE GOTO 1840
1870 INPUT "combined magnetic, processing & kinematic data file name"; E$
1880 ON ERROR GOTO 1890: GOTO 1900
1890 RESUME 1840
1900 OPEN "O", #1, E$
1910 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV,
FL, FT, FV, DT, IT
1920 WRITE #1, NS, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT, M$, P$, K$
1930 CLOSE
1940 GOTO 2110
1950 INPUT "combined magnetic, processing & kinematic data file name"; E$
1960 ON ERROR GOTO 1970: GOTO 1980
1970 RESUME 1950
1980 OPEN "I", #1, E$
1990 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV,
FL, FT, FV, DT, IT
2000 INPUT #1, NS, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT, M$, P$, K$
2010 CLOSE
2020 DECR = DEC * CON: DIPR = DIP * CON
2030 INPUT "required false alarm rate in false alarms per hour"; FAR
2040 PF = FAR * IT / 3600: REM false alarm probability
2050 Y = PF: IF PF > .5 THEN Y = 1 - Y: REM inverse normal approximation
2060 Y = SQR(LOG(1 / Y / Y))
2070 Y = Y - (I1 + Y * (I2 + I3 * Y)) / (1 + Y * (I4 + Y * (I5 + I6 * Y)))
2080 IF PF < .5 THEN Y = -Y
2090 ZP = -Y
2100 CHI = NS * (1 - 2 / 9 / NS + ZP * SQR(2 / 9 / NS)) ^ 3: REM inverse
chi-square approximation
2110 INPUT "magnetic noise in gamma"; SIG
2120 INPUT "maximum lateral range in meters"; LRM
2130 INPUT "lateral range step in meters"; ST
2140 IF ST <= LRM THEN GOTO 2180
2150 PRINT : PRINT "maximum step is " + STR$(LRM) + " meters": PRINT :
GOTO 2130
2180 NC = 2 * INT(LRM / ST) + 1: REM number of lateral range function values
2190 IF NC <= 71 THEN GOTO 2210
```

```
2200 PRINT : PRINT "minimum step is " + STR$(LRM / 35) + " meters": PRINT
: GOTO 2130
2210 'start of mods
2220 INPUT "number of buoys in the field"; NB
2230 INPUT "duration of the search in hours"; SD
2240 INPUT "search area in square nautical miles"; A: A = A * 3429904'conversion
to square meters
2250 INPUT "fraction of buoys that can be monitored at one time"; FR
2260 IF FR <= 1 THEN GOTO 2280
2270 PRINT : PRINT "fraction must be less than or equal to 1": GOTO 2250
2280 ATZ = AT
2290 FOR ZM = ATMIN TO ATZ STEP ATINT: REM loop for depth
2300 AT = ZM: AREA1 = 0: AREA2 = 0
2310 FOR ZQ = 0 TO 150 STEP 30: REM loop for course
2320 CT = ZQ
2330 PDCCP = 0!: PDSLP = 0!
2340 APDCCT = 0!: APDSLT = 0!
2350 CMR = CM * CON: CTR = CT * CON
2360 WXK = VMK * SIN(CMR) - VTK * SIN(CTR)
2370 WYK = VMK * COS(CMR) - VTK * COS(CTR)
2380 Z = AM + AT: REM vertical separation (- for magnetometer below target)
2390 X = WXK: Y = WYK: GOSUB 4430
2400 CR = T: C = CR / CON: REM magnetometer relative course
2410 WK = R: REM magnetometer relative speed
2420 CMR = CM * CON: CTR = CT * CON: CR = C * CON
2430 MCMR = (CMR - DECR): REM magnetometer magnetic course in radians
2440 MCTR = (CTR - DECR): REM target magnetic course in radians
2450 ILM = KL * HE * COS(DIPR) * COS(MCTR): ITM = -KT * HE * COS(DIPR)
* SIN(MCTR)
2460 IVM = KV * HE * SIN(DIPR)
2470 DMX = (PLM + ILM) * SIN(MCTR) + (PTM + ITM) * COS(MCTR)
2480 DMY = (PLM + ILM) * COS(MCTR) - (PTM + ITM) * SIN(MCTR)
2490 DMV = PVM + IVM
2500 X = DMX: Y = DMY
2510 GOSUB 4430
2520 OMLR = T: REM dipole azimuth relative to magnetic north
2530 X = DMV: Y = R
2540 GOSUB 4430
2550 DM = R: OMR = T: REM dipole depression angle
2560 OML = OMLR / CON: OM = OMR / CON
2570 AIT = DT * NS: REM adjusted integration time
2585 W = WK * KON: REM magnetometer relative speed in meters/second
2590 DS = W * DT: REM distance between samples on the relative track
2600 X0 = -(NC - 1) / 2 * ST
2610 FOR I = 0 TO NC - 1
2620 X0(I) = X0
2630 X = X0: Y = Z
2640 GOSUB 4430
2650 R0 = R: R0(I) = R: REM target slant range at CPA in meters
2660 DELR = T: REM target depression angle complement at CPA in radians
```

```
2670 IF R0 = 0 THEN GOTO 3000: REM zero lateral range and vertical separation
2680 DMF = DM / 10 / R0 ^ 3: REM dipole moment factor
2690 CMR = CR - DECR: REM target relative magnetic course
2700 L = COS(OMR) * COS(CMR - OMLR)
2710 M = COS(DELR) * COS(OMR) * SIN(CMR - OMLR) - SIN(DELR) *
SIN(OMR)
2720 N = -SIN(DELR) * COS(OMR) * SIN(CMR - OMLR) - COS(DELR) *
SIN(OMR)
2730 L1 = COS(DIPR) * COS(CMR)
2740 M1 = COS(DELR) * COS(DIPR) * SIN(CMR) - SIN(DELR) * SIN(DIPR)
2750 N1 = -SIN(DELR) * COS(DIPR) * SIN(CMR) - COS(DELR) * SIN(DIPR)
2760 A2 = 2 * L * L1 - M * M1 - N * N1: REM Anderson Function Coefficient
2770 A1 = 3 * (N * L1 + L * N1): REM Anderson Function Coefficient
2780 A0 = 2 * N * N1 - L * L1 - M * M1: REM Anderson Function Coefficient
2790 SUM = 0: HMAX = 0: HMIN = 0
2800 FOR J = 0 TO NS - 1
2810 S = (J - (NS - 1) / 2) * DS: BA = S / R0: REM Anderson Function Argument
2820 AF = 1 / (1 + BA * BA) ^ 2.5: REM Anderson Function Factor
2830 HSF = (A2 * BA * BA + A1 * BA + A0) * AF: REM magnetic signal factor
2840 HS(I, J) = DMF * HSF: REM magnetic signal value
2850 IF HS(I, J) > HMAX THEN HMAX = HS(I, J)
2860 IF HS(I, J) < HMIN THEN HMIN = HS(I, J)
2870 SUM = SUM + HSF * HSF
2880 NEXT J
2890 H(I) = HMAX - HMIN
2900 K(I) = SQR(SUM)
2910 VV = -ZP + DMF * SQR(SUM) / SIG
2920 LAM = DMF * DMF * SUM / (SIG * SIG): AA = NS + LAM: BB = 1 + LAM
/ (NS + LAM)
2930 ZN = -SQR(2 * CHI / BB) + SQR(2 * AA / BB - 1): X1 = VV
2940 GOSUB 4490
2950 IF Y1 > 1 THEN Y1 = 1
2960 PDCC(I) = Y1: X1 = ZN
2970 GOSUB 4490
2980 IF Y1 > 1 THEN Y1 = 1
2990 PDSL(I) = Y1
3000 X0 = X0 + ST
3010 REM this section calculates the sweep width as the area under
3020 'the lateral range curve
3030 IF I > 0 THEN
3040 IF PDCCP <= PDCC(I) THEN
3050    AREAI1 = (PDCCP * ST) + ((ST * ABS(PDCC(I) - PDCCP)) / 2)
3060 ELSE
3070    AREAI1 = (PDCC(I) * ST) + ((ST * ABS(PDCCP - PDCC(I))) / 2)
3080 END IF
3090 REM APDCCT & APDSLT are the areas under their respective lateral
3100 'range curves
3110 APDCCT = APDCCT + AREAI1
3120 IF PDSLP <= PDSL(I) THEN
3130    AREAI2 = (PDSLP * ST) + ((ST * ABS(PDSL(I) - PDSLP)) / 2)
```

```
3140 ELSE
3150    AREAI2 = (PDSL(I) * ST) + ((ST * ABS(PDSLP - PDSL(I))) / 2)
3160 END IF
3170 APDSLT = APDSLT + AREAI2
3180 END IF
3190 PDCCP = PDCC(I)
3200 PDSLP = PDSL(I)
3210 NEXT I
3220 C = C / 360: C = (C - INT(C)) * 360
3230 IF C < 0 THEN C = 360 + C
3240 OML = (OML + DEC) / 360: OML = (OML - INT(OML)) * 360
3250 IF OML < υ THEN OML = 360 + OML
3260 REM AREA1&2 sums the sweep width over all courses for a given depth
3270 AREA1 = AREA1 + APDCCT: AREA2 = AREA2 + APDSLT
3280 NEXT ZQ
3290 AREA1T(COUNT) = AREA1 / 6 'calculating the average sweep width
3300 REM PDET1T() calculates the buoy field probability of detection
3310 'for a crosscorrelation detector and PDET2T() for a square law detector
3320 PDET1T(COUNT) = 1 - EXP(-NB * FR * AREA1T(COUNT) * VTK * 1852 *
SD / A)
3330 AREA2T(COUNT) = AREA2 / 6
3340 PDET2T(COUNT) = 1 - EXP(-NB * FR * AREA2T(COUNT) * VTK * 1852 *
SD / A)
3350 COUNT = COUNT + 1
3360 NEXT ZM
3370 PRINT : A$ = "a"
3390 INPUT "generate a program data file (y/n)"; A$
3400 IF A$ = "Y" OR A$ = "y" THEN GOTO 3420
3410 IF A$ = "N" OR A$ = "n" THEN GOTO 3650 ELSE GOTO 3390
3420 INPUT "program data file name"; D$
3430 ON ERROR GOTO 3440: GOTO 3450
3440 RESUME 3420
3450 OPEN "O", #1, D$
3460 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV,
FL, FT, FV, DT, IT, AIT
3470 WRITE #1, NS, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT, COUNT, FAR,
PF, SIG, ST
3480 WRITE #1, NB, SD, A, FR, LRM, DS, NC, M$, P$, K$, E$
3490 FOR I = 0 TO COUNT
3500 WRITE #1, AREA1T(I), AREA2T(I)
3510 NEXT I
3520 CLOSE
3530 GOTO 3650
3540 INPUT "program data file name"; D$
3550 ON ERROR GOTO 3560: GOTO 3570
3560 RESUME 3540
3570 OPEN "I", #1, D$
3580 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV,
FL, FT, FV, DT, IT, AIT
```

```
3590 INPUT #1, NS, CM, VMK, AM, CT, VTK, AT, ATMIN, ATINT, COUNT, FAR,
PF, SIG, ST
3600 INPUT #1, NB, SD, A, FR, LRM, DS, NC, M$, P$, K$, E$
3610 FOR I = 0 TO COUNT
3620 INPUT #1, AREA1T(I), AREA2T(I)
3630 NEXT I
3640 CLOSE
3650 PRINT : A$ = "a"
3660 INPUT "print encounter parameter values (y/n)"; A$
3670 IF A$ = "Y" OR A$ = "y" THEN GOTO 3690
3680 IF A$ = "N" OR A$ = "n" THEN GOTO 4140 ELSE GOTO 3660
3690 LPRINT
3700 LPRINT "program file name                                          " + N$
3710 LPRINT "program data file name                                     " + D$
3720 LPRINT "magnetic data file name                                    " + M$
3730 LPRINT "processing data file name                                  " + P$
3740 LPRINT "kinematic data file name                                   " + K$
3750 LPRINT "combined magnetic, processing & kinematic data file name " + E$
3760 LPRINT "encounter latitude (decimal degrees)                    "; SPC(2); LAT
3770 LPRINT "encounter longitude (decimal degrees)                   "; SPC(2); LNG
3780 LPRINT "encounter variation (decimal degrees)                   "; SPC(2); DEC
3790 LPRINT "encounter dip angle (decimal degrees)                   "; SPC(2); DIP
3800 LPRINT "encounter magnetic field intensity (gamma)              "; SPC(2); HE
3810 LPRINT "permanent longitudinal moment (oersted-cm3)               "; SPC(2);
PLM
3820 LPRINT "permanent transverse moment (oersted-cm3)                 "; SPC(2);
PTM
3830 LPRINT "permanent vertical moment (oersted-cm3)                   "; SPC(2);
PVM
3840 LPRINT "target displacement (tons)                              "; SPC(2); WT
3850 LPRINT "target longitudinal permeability coefficient            "; SPC(2); KL
3860 LPRINT "target transverse permeability coefficient              "; SPC(2); KT
3870 LPRINT "target vertical permeability coefficient                "; SPC(2); KV
3880 LPRINT "target longitudinal permeability factor                 "; SPC(2); FL
3890 LPRINT "target transverse permeability factor                   "; SPC(2); FT
3900 LPRINT "target vertical permeability factor                     "; SPC(2); FV
3910 LPRINT "sampling period (seconds)                               "; SPC(2); DT
3920 LPRINT "integration time (seconds)                              "; SPC(2); IT
3930 LPRINT "adjusted integration time (seconds)                     "; SPC(2); AIT
3940 LPRINT "number of samples per encounter                         "; SPC(2); NS
3950 LPRINT "magnetometer depth (meters)                             "; SPC(2); AM
3960 LPRINT "target speed (knots)                                    "; SPC(2); VTK
3970 LPRINT "maximum target depth (meters)                           "; SPC(2); AT
3980 LPRINT "minimum target depth (meters)                             "; SPC(2);
ATMIN
3990 LPRINT "depth interval (meters)                               "; SPC(2); ATINT
4000 LPRINT "distance between samples on the relative track (meters)"; SPC(2); DS
4010 LPRINT "false alarm rate (false alarms per hour)                "; SPC(2); FAR
4020 LPRINT "false alarm probability                                 "; SPC(2); PF
4030 LPRINT "magnetic noise (gamma)                                  "; SPC(2); SIG
```

```
4040 LPRINT "maximum lateral range (meters)                    "; SPC(2); LRM
4050 LPRINT "lateral range step (meters)                       "; SPC(2); ST
4060 LPRINT "number of lateral range function values           "; SPC(2); NC
4070 LPRINT "number of buoys in the field                      "; SPC(2); NB
4080 LPRINT "duration of the search (hours)                    "; SPC(2); SD
4090 LPRINT "search area in square nuatical miles              "; SPC(2); A
4100 LPRINT "fraction of buoys that can be monitored at one time "; SPC(2); FR
4110 FOR I = 0 TO 21
4120 LPRINT
4130 NEXT I
4140 PRINT : A$ = "a"
4150 INPUT "print buoy field probability of detection values (y/n)"; A$
4160 IF A$ = "Y" OR A$ = "y" THEN GOTO 4180
4170 IF A$ = "N" OR A$ = "n" THEN GOTO 4260 ELSE GOTO 4150
4180 LPRINT D$; " buoy field probability of detection values"
4190 LPRINT : LPRINT
4200 LPRINT "depth      p(cc)        p(sl)"
4210 LPRINT "meters"
4220 LPRINT
4230 FOR I = 0 TO COUNT - 1
4240 LPRINT ATINT * I; TAB(10); PDET1T(I); TAB(24); PDET2T(I)
4250 NEXT I
4260 PRINT : A$ = "a"
4270 INPUT "print average sweep width values (y/n)"; A$
4280 IF A$ = "Y" OR A$ = "y" THEN GOTO 4300
4290 IF A$ = "N" OR A$ = "n" THEN GOTO 4380 ELSE GOTO 4270
4300 LPRINT D$; " average sweep width values"
4310 LPRINT : LPRINT
4320 LPRINT "depth      avg(cc)       avg(sl)"
4330 LPRINT "meters     meters         meters"
4340 LPRINT
4350 FOR I = 0 TO COUNT - 1
4360 LPRINT ATINT * I; TAB(10); AREA1T(I); TAB(24); AREA2T(I)
4370 NEXT I
4380 PRINT : A$ = "a"
4390 INPUT "continue to use the program (y/n)"; A$
4400 IF A$ = "Y" OR A$ = "y" THEN GOTO 10
4410 IF A$ = "N" OR A$ = "n" THEN GOTO 4420 ELSE GOTO 4390
4420 END
4430 R = SQR(X * X + Y * Y): REM rectangular to polar conversion
4440 IF R = 0 THEN T = 0: RETURN
4450 IF ABS(X / R) = 1 THEN Q = SGN(X) * (PI / 2) ELSE Q = ATN(X / R /
SQR(1 - X * X / R / R))
4460 IF ABS(Y / R) = 1 THEN T = (PI / 2) * (1 - SGN(Y)) ELSE T = (PI / 2) -
ATN(Y / R / SQR(1 - Y * Y / R / R))
4470 IF Q < 0 THEN T = 2 * PI - T
4480 RETURN
4490 Y1 = X1: IF X1 < 0 THEN Y1 = -Y1: REM normal approximation
4500 G = 1 / (1 + Q0 * Y1)
```

```
4510 Y1 = EXP(-Y1 * Y1 / 2) / SQR(2 * PI) * G * (Q1 + G * (Q2 + G * (Q3 + G
* (Q4 + G * Q5))))
4520 IF X1 >= 0 THEN Y1 = 1 - Y1
4530 RETURN
```

LIST OF REFERENCES

1. Naval Postgraduate School Report NPS71-88-001 (Revised), *A Program to Compute Magnetic Anomaly Detection Probabilities*, by R. N. Forrest, October 1988

2.Naval Postgraduate School Report NPS71-87-001 (Third Revision), *Notes on Search, Detection and Localization Modeling*, by R. N. Forrest, April 1987

3.Naval Postgraduate School Report NPS55-79-06 (Revised), *The Search Effectiveness of MAD*, by R. N. Forrest, February 1979 (Revised September 1980)

4.Washburn, A. R., *Search and Detection*, 2d ed., Operations Research Society of America, 1989

5.*Jane's Fighting Ships 1989-90*, 92d ed., Jane's Information Services, 1989

6.Texas Instruments Incorporated Report C2-61009-2, *MAD Signal Processing Study*, 1 April 1961

7.Naval Postgraduate School Report NPS55-77-19, *Magnetic Anomaly Detection Models*, by R. N. Forrest, April 1988

INITIAL DISTRIBUTION LIST

1.	Defense Techinical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Attn: Code 0142 Naval Postgraduate School Monterey, CA 93943-5000	2
3.	Commander Submarine Development Squadron 12 Naval Submarine Base, New London Groton, CT 06349	1
4.	Commander Surface Warfare Development Group Naval Amphibious Base, Little Creek Norfolk, VA 23521	1
5.	Naval Air Development Center Johnsville, PA 18974	1
6.	Naval Surface Weapons Center White Oak Silver Spring, MD 20910	1
7.	Naval Underwater Systems Center Newport, RI 02840	1
8.	Naval Underwater Systems Center New London, CT 06320	1
9.	Naval Technical Intelligence Center 4301 Suitland Road Washington, DC 20390	1
10.	Naval Research Laboratory Washington, DC 20375	1
11.	Center for Naval Analysis 4401 Ford Avenue P. O. Box 16268 Alexandria, VA 22302-0268	1

12.	Naval Ocean Systems Center San Diego, CA 92132	1
13.	Naval Postgraduate School Monterey, CA 93943-5000 Code 55Fo	10
14.	Naval Postgraduate School Monterey, CA 93943-5000 Code 55Er	1
15.	Naval Postgraduate School Monterey, CA 93943-5000 Code 55Wa	1
16.	LT Phillip C. Pardue Department Head Class 114 Surface Warfare Officers School Command Newport, RI 02841-5012	1